Hints and answers from 5.2 and 5.1

5.2 #4) First hint – we didn't do trig functions yet, so I shouldn't have assigned this ☹ Sorry !!

5.2 #7) Locate critical numbers and identify which critical numbers are stationary points (derivative is 0 as opposed to undefined). I want to be the best and so I'll evaluate these stationary points for relative maxima or minima as well! **(a)** $f(x) = x^3 + 3x^2 - 9x + 1$

Step 1: Take derivative

$$f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x+3)(x-1)$$

Step 2: Set derivative equal to 0

$3(x+3)(x-1) = 0 \Rightarrow x = -3, +1$ are the critical numbers – and also stationary points.

The derivative is always defined so there are no additional critical numbers.

Step 3: We'll use the first derivative test (remember – in finding relative extrema the 1st derivative test always works). Construct a sign chart….

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-3 +1

+ + + + 0 - - - - - - - - - 0 + + + +

Notice that at x = -3, the 1st derivative changes from positive (original function increasing) to negative (original function decreasing). Therefore a relative maximum occurs at x = -3.
Notice also that at x = +1, the 1st derivative changes from a negative (original function decreasing) to a positive (original function increasing). Therefore a relative minimum occurs at x = +1.

And we are done ☺

Step 3 - alternative: We could also use the second derivative test.

$f''(x) = 6x + 6$

$f''(-3) = 6(-3) + 6 = -12$ Since $f''(-3) < 0$, a rel. max. occurs there. Since $f''(-3) > 0$

$f''(1) = 6(1) + 6 = +12$

that's a rel. min!

(b) $f(x) = x^4 - 6x^2 - 3$
$f'(x) = 4x^3 - 12x = 4x\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)$ and proceed like above.

5.1 #15) $f(x) = 3x^4 - 4x^3$. So … $f'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$. On my sign chart I want all the x values that make the 1st derivative zero or undefined.

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0 +1

- - - - - - 0 - - - - - - - - - 0 + + + +

And … because of the multiplicity of x^2, the 1st derivative is negative on $(-\infty, 1)$ and positive on $(1, \infty)$.

Therefore: **(a)** $[1, \infty)$ **(b)** $(-\infty, 1]$ To answer (c)-(e) we need to look at the second derivative:

$f''(x) = 36x^2 - 24x = 12x(3x - 2)$. And a sign chart for second derivative …

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0 $+\frac{2}{3}$

+ + + + 0 - - - - - - - - - 0 + + + +

And now … **(c)** concave up: $\left(\frac{2}{3},\infty\right)$ **(d)** concave down: $\left(-\infty,\frac{2}{3}\right)$ **(e)** inflection point at $x = 0$ and 2/3.